Thus, the statements in Sec. I remain valid, but the rate of approach to the asymptotic limit, discussed in Sec. II, must be corrected by the terms of order $1/\ln^2\nu$. Lacking any method for estimating the size of these terms, we must rely on experiment to determine the energy at which asymptotic forms can be used. In particular, one can apply the test proposed in this paper: Measure $\mathrm{Re}f^{(-)}/\mathrm{Im}f^{(-)}$ at several energies to check whether it is constant. If it is, its ratio is simply $\tan(\pi\alpha_\rho/2)$, as in the case of no branch cuts.

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# Classical Radiation Recoil. II*

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The angular momentum carried off by electromagnetic or gravitational waves is evaluated by computing the torque exerted by the radiation on the emitting system. The lowest order secular effect in the electromagnetic case is proportional to the vector product of the first and second time derivatives of the electric dipole. In the gravitational case, the lowest order term is proportional to the antisymmetrized product of the second and third time derivatives of the mass quadrupole.

IT is well known that angular momentum can be carried off by electromagnetic waves[1] and presumably also by gravitational waves. The purpose of this paper is to apply the previously developed technique[2] to a direct calculation of the torque exerted on the emitting system by its own electromagnetic or gravitational radiation. This method seems physically more meaningful than discussions about what is happening at spatial infinity.

The torque is given by

$$N^k=\epsilon^{kmn}\int x_m F_n dV, \tag{1}$$

where $F_n$ is the force density. In the electromagnetic case, we have

$$F_n=(A_{\alpha n}-A_{n\alpha})J^\alpha. \tag{2}$$

With the help of the continuity equation $J_\alpha{}^\alpha=0$, it follows that

$$N^k=\epsilon^{kmn}\int(x_m A_{\alpha n}J^\alpha+A_n J_m)dV, \tag{3}$$

and substitution of (A12), (A13), and repeated use of (A14) readily lead to

$$N^k=\tfrac{4}{3}\epsilon^{kmn}\,{}^1D_m\,{}^2D_n, \tag{4}$$

in agreement with the usual result.[1]

In the gravitational case, we have[2]

$$F_n=\tfrac{1}{2}(V_{n\alpha\beta}+V_{n\beta\alpha}+\tfrac{1}{2}g_{\alpha\beta}V^\gamma{}_{\gamma n}-V_{\alpha\beta n})T^{\alpha\beta}. \tag{2'}$$

With the help of the dynamic equation $T^{\alpha\beta}{}_\beta=0$, it follows that

$$N^k=\epsilon^{kmn}\int[\tfrac{1}{2}x_m(\tfrac{1}{2}g_{\alpha\beta}V^\gamma{}_{\gamma n}-V_{\alpha\beta n})T^{\alpha\beta}-V_{n\alpha}T^\alpha{}_m]dV, \tag{3'}$$

and substitution of (A12′), (A13′), and repeated use of (A14′) and (A14″) readily lead to

$$N^k=\tfrac{2}{5}\epsilon^{kmn}\,{}^3Q_{mj}\,{}^2Q_{jn}. \tag{4'}$$

It would be interesting to rederive this result by computing the angular momentum flow, in the wave zone, of the gravitational radiation, as recently done by Papapetrou for the linear momentum flow.[3] This seems, however, to be an extremely tedious calculation, much more difficult than in the electromagnetic case.[1]

It should also be noted that radiation recoil is a secular (cumulative) effect. It might, therefore, be easier to detect than the instantaneous gravitational radiation energy flow.

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